

Ex) $\frac{1}{2}, \sqrt{5}, -\sqrt{5}, \sqrt{3}i, -\sqrt{3}i$

$f(x) = 2x^5 - x^4 - 4x^3 + 2x^2 - 30x + 15$

Possible Rational: $\frac{\pm 1, \pm 3, \pm 5, \pm 15}{\pm 1, \pm 2}$ $\frac{1}{2},$

$\pm 1, \pm 3, \pm 5, \pm 15, \pm \frac{1}{2}$

$\pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

Mar 31-8:04 AM

Ex) $f(x) = 3x^4 - 11x^3 + 10x - 4$

$\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

$\boxed{-1, \frac{2}{3}, 2+\sqrt{2}, 2-\sqrt{2}}$

Mar 31-8:20 AM

55-64 p. 340

56) $f(x) = x^3 - x^2 - 3x + 3$

$\frac{\pm 1, \pm 3}{\pm 1} = \pm 1, \pm 3$

$\boxed{1, \sqrt{3}, -\sqrt{3}}$

$\begin{array}{r|rrrr} 1 & 1 & -1 & -3 & 3 \\ & \downarrow & 1 & 0 & -3 \\ \hline & 1 & 0 & -3 & 0 \end{array}$

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm \sqrt{3}$

Mar 31-8:27 AM

58) $f(x) = x^3 - 2x + 4$

$\boxed{-2, 1+i, 1-i}$

$\frac{\pm 1, \pm 2, \pm 4}{\pm 1} = \pm 1, \pm 2, \pm 4$

$\begin{array}{r|rrrr} -2 & 1 & 0 & -2 & 4 \\ & \downarrow & -2 & 4 & -4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$

$x^2 - 2x + 2 = 0$

$a=1, b=-2, c=2$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$

Mar 31-8:33 AM

55) $f(x) = x^3 + 3x^2 - 2x - 6$

$\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1} = \pm 1, \pm 2, \pm 3, \pm 6$

$\boxed{-3, \sqrt{2}, -\sqrt{2}}$

$\begin{array}{r|rrrr} -3 & 1 & 3 & -2 & -6 \\ & \downarrow & -3 & 7 & 6 \\ \hline & 1 & 0 & 7 & 0 \end{array}$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm \sqrt{2}$

Mar 31-8:37 AM

Example:

$f(x) = 3x^4 - 11x^3 + 10x - 4$

$\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$

$\boxed{-1, \frac{2}{3}, 2+\sqrt{2}, 2-\sqrt{2}}$

Mar 31-8:53 AM

$$5b) f(x) = x^3 - x^2 - 3x + 3$$

$$\frac{\pm 1, \pm 3}{\pm 1} = \pm 1, \pm 3 \quad \begin{array}{r} 1 \mid 1 \quad -1 \quad -3 \quad 3 \\ \downarrow \quad 1 \quad 0 \quad -3 \\ 1 \quad 0 \quad -3 \quad 0 \end{array}$$

$$\boxed{1, \sqrt{3}, -\sqrt{3}}$$

$$x^2 - 3 = 0$$

$$\sqrt{x^2 - 3} = \sqrt{3}$$

$$x = \pm \sqrt{3}$$

Mar 31-9:24 AM

$$5b) f(x) = x^3 - 2x + 4$$

$$\frac{\pm 1, \pm 2, \pm 4}{\pm 1} = \pm 1, \pm 2, \pm 4 \quad \begin{array}{r} -2 \mid 1 \quad 0 \quad -2 \quad 4 \\ \downarrow \quad -2 \quad 4 \quad -4 \\ 1 \quad -2 \quad 2 \quad 0 \end{array}$$

$$\boxed{-2, 1+i, 1-i}$$

$$x^2 - 2x + 2 = 0$$

$$\hat{a}=1 \quad \hat{b}=-2 \quad \hat{c}=2$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

Mar 31-9:31 AM

$$6b) f(x) = 2x^3 + 7x^2 + 2x - 8$$

$$\frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$$

$$\begin{array}{r} -2 \mid 2 \quad 7 \quad 2 \quad -8 \\ \downarrow \quad -4 \quad -6 \quad 8 \\ 2 \quad 3 \quad -4 \quad 0 \end{array}$$

$$\boxed{-2, \frac{-3+\sqrt{41}}{4}, \frac{-3-\sqrt{41}}{4}}$$

$$2x^2 + 3x - 4 = 0$$

$$a=2 \quad b=3 \quad c=-4$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-4)}}{2(2)} = \frac{-3 \pm \sqrt{41}}{4}$$

Mar 31-9:38 AM

$$6b) f(x) = 3x^4 - 4x^3 + x^2 + 6x - 2$$

$$\frac{\pm 1, \pm 2}{\pm 1, \pm 3} = \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$$

$$\begin{array}{r} -1 \mid 3 \quad -4 \quad 1 \quad 6 \quad -2 \\ \downarrow \quad -3 \quad 7 \quad -8 \quad 2 \\ \frac{1}{3} \mid 3 \quad -7 \quad 8 \quad -2 \quad 0 \\ \downarrow \quad 1 \quad -2 \quad 2 \\ 3 \quad -6 \quad 6 \quad 0 \end{array}$$

$$\boxed{1, \frac{1}{3}, 1+i, 1-i}$$

$$3x^2 - 6x + 6 = 0$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(3)(6)}}{2(3)} = \frac{6 \pm \sqrt{-36}}{6} = \frac{6 \pm 6i}{6} = 1 \pm i$$

Mar 31-9:44 AM

$$6b) f(x) = x^4 + 5x^3 - 27x^2 + 31x - 10$$

$$\frac{\pm 1, \pm 2, \pm 5, \pm 10}{\pm 1, \pm 2, \pm 5, \pm 10} = \pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{5}, \pm \frac{1}{10}$$

$$\begin{array}{r} 1 \mid 1 \quad 5 \quad -27 \quad 31 \quad -10 \\ \downarrow \quad 1 \quad 6 \quad -21 \quad 10 \\ 2 \mid 1 \quad 6 \quad -21 \quad 10 \quad 0 \\ \downarrow \quad 2 \quad 16 \quad -10 \\ 1 \quad 8 \quad -5 \quad 0 \end{array}$$

$$\boxed{1, 2, -4+\sqrt{21}, -4-\sqrt{21}}$$

$$x^2 + 8x - 5 = 0$$

$$a=1 \quad b=8 \quad c=-5$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-5)}}{2(1)} = \frac{-8 \pm \sqrt{84}}{2} = \frac{-8 \pm 2\sqrt{21}}{2} = -4 \pm \sqrt{21}$$

Mar 31-9:50 AM

$$6c) f(x) = 3x^4 - 11x^3 + 10x^2 - 4$$

$$\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 3} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}$$

$$\begin{array}{r} -1 \mid 3 \quad -11 \quad 0 \quad 10 \quad -4 \\ \downarrow \quad -3 \quad 14 \quad -14 \quad 4 \\ \frac{2}{3} \mid 3 \quad -14 \quad 14 \quad -4 \quad 0 \\ \downarrow \quad 2 \quad -8 \quad 4 \\ 3 \quad -12 \quad 6 \quad 0 \end{array}$$

$$\boxed{-1, \frac{2}{3}, 2+\sqrt{2}, 2-\sqrt{2}}$$

Mar 31-11:06 AM

$$58) f(x) = x^3 - 2x + 4$$

$$\pm 1, \pm 2, \pm 4$$

$$\boxed{-2, 1+i, 1-i}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -2 & 4 \\ & \downarrow & -2 & 4 & -4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$$x^2 - 2x + 2 = 0$$

$$a=1 \quad b=-2 \quad c=2$$

$$1 \pm i = \frac{2 \pm 2i}{2} = x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2}$$

$$60) f(x) = 2x^3 + 7x^2 + 2x - 8$$

$$\pm 1, \pm 2, \pm 4, \pm 8$$

$$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$$

$$\boxed{-2, -\frac{3+\sqrt{41}}{4}, -\frac{3-\sqrt{41}}{4}}$$

$$\begin{array}{r|rrrr} -2 & 2 & 7 & 2 & -8 \\ & \downarrow & -4 & -6 & 8 \\ \hline & 2 & 3 & -4 & 0 \end{array}$$

$$2x^2 + 3x - 4 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-4)}}{2(2)} = \frac{-3 \pm \sqrt{41}}{4}$$

Mar 31-12:54 PM

Mar 31-1:00 PM

$$62) f(x) = 3x^4 - 4x^3 + x^2 + 6x - 2$$

$$\pm 1, \pm 2$$

$$\pm 1, \pm 3$$

$$\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$$

$$\boxed{-1, \frac{1}{3}, 1+i, 1-i}$$

$$x = \frac{6 \pm \sqrt{(-4)^2 - 4(3)(6)}}{2(3)} = \frac{6 \pm 6i}{6} = 1 \pm i$$

$$\begin{array}{r|rrrrr} -1 & 3 & -4 & 1 & 6 & -2 \\ & \downarrow & -3 & 7 & -8 & 2 \\ \hline & 3 & -7 & 8 & -2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & -7 & 8 & -2 & 0 \\ & \downarrow & 1 & -2 & 2 & \\ \hline & 3 & -6 & 6 & 0 & \end{array}$$

$$3x^2 - 6x + 6 = 0$$

$$3x^2 - 6x + 6 = 0$$

Mar 31-1:08 PM

Mar 31-1:08 PM

$$64) f(x) = x^4 + 5x^3 - 27x^2 + 31x - 10$$

$$\pm 1, \pm 2, \pm 5, \pm 10$$

$$\boxed{1, 2, -4+\sqrt{21}, -4-\sqrt{21}}$$

$$= \frac{-4 \pm \sqrt{21}}{2}$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(-5)}}{2(1)} = \frac{-8 \pm \sqrt{71}}{2}$$

$$\begin{array}{r|rrrrr} 1 & 1 & 5 & -27 & 31 & -10 \\ & \downarrow & 1 & 6 & -21 & 10 \\ \hline & 1 & 6 & -21 & 10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & 6 & -21 & 10 & 0 \\ & \downarrow & 2 & 16 & -10 & \\ \hline & 1 & 8 & -5 & & \end{array}$$

$$x^2 + 8x - 5 = 0$$

Mar 31-1:19 PM